

# Non-planar ABJM Theory and Integrability

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## Abstract

Using an effective vertex method we explicitly derive the two-loop dilatation generator of ABJM theory in its  $SU(2) \times SU(2)$  sector, including all non-planar corrections. Subsequently, we apply this generator to a series of finite length operators as well as to two different types of BMN operators. As in  $\mathcal{N} = 4$  SYM, at the planar level the finite length operators are found to exhibit a degeneracy between certain pairs of operators with opposite parity – a degeneracy which can be attributed to the existence of an extra conserved charge and thus to the integrability of the planar theory. When non-planar corrections are taken into account the degeneracies between parity pairs disappear hinting the absence of higher conserved charges. The analysis of the BMN operators resembles that of  $\mathcal{N} = 4$  SYM. Additional non-planar terms appear for BMN operators of finite length but once the strict BMN limit is taken these terms disappear.

# 1 Introduction

Integrability has been the driving force behind the recent years' progress in the study of the spectral problem of the  $AdS_5/CFT_4$  correspondence [1, 2, 3]. Integrability is conjectured to hold in all sectors to all loop orders [2, 4] and impressive tests involving quantities extrapolating from weak to strong coupling have been performed [5, 6, 7, 8].

Recently a novel explicit example of a gauge/string duality of type  $AdS_4/CFT_3$  has emerged [9] and one could hope that integrability would play an equally important role there. So far in the  $AdS_4/CFT_3$  correspondence integrability is at a much less firm setting. The gauge theory dilatation generator has been proved to be integrable in the scalar sector at leading two-loop order [10, 11] and the string theory has been proved to be classically integrable in certain subsectors [12, 13, 14]. Investigations probing integrability at the quantum level of the string theory have been carried out in various regimes such as the BMN limit [15, 16, 17], the giant magnon regime [17, 18] and the near BMN and near flat-space limits [19, 20]. There exist conjectures about integrability of the full  $AdS_4/CFT_3$  system in all sectors to all loops [21] and a number of tests have come out affirmative [19, 22, 23, 24] but certain problems still seem to require resolution [24].

The spectral information only constitutes one part of the information encoded in the gauge and string theory. Eventually, one would like to go beyond the spectral problem and study interacting string theory respectively non-planar gauge theory. A widespread expectation is that integrability cannot persist beyond the planar limit. In reference [2] a way to characterize and quantify the deviation from integrability was presented for  $\mathcal{N} = 4$  SYM. In this case one observed at the planar level some a priori unexpected degeneracies in anomalous dimensions between certain pairs of operators with opposite parity. These degeneracies could be explained by the existence of an extra conserved charge and thus eventually by the integrability of the theory. When non-planar corrections were taken into account these degeneracies were found to disappear. Notice, however, that the degeneracies observed at planar one-loop order persisted when planar higher loop corrections were taken into account. This observation was in fact the seed that led to the conjecture about all loop integrability of  $\mathcal{N} = 4$  SYM [2].

In the present paper we will study non-planar corrections to  $\mathcal{N} = 6$  superconformal Chern–Simons–matter theory, the three-dimensional field theory entering the  $AdS_4/CFT_3$  correspondence, in order to investigate whether one observes a similar lifting of spectral degeneracies related to integrability when one goes beyond the planar level. Our investigations will be carried out in the  $SU(2) \times SU(2)$  sector at two-loop level and will thus not rely on or involve any conjectures.

Using a method based on effective vertices we will derive the full two-loop dilatation generator in this sector involving all non-planar corrections. For short operators the action of this dilatation generator can easily be written down, resulting in a mixing matrix of low dimension which can be diagonalized explicitly.<sup>1</sup> Another type of operators for which the mixing matrix can easily be written down are BMN-type operators [26] which contain a large (infinite) number of background fields and a small (finite) number of excitations. We will look into the nature of the BMN quantum mechanics [27] of

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<sup>1</sup>For  $\mathcal{N} = 4$  SYM, explicit diagonalization at the non-planar level for a range of operators of this type was carried out in [2], see also [25].

$\mathcal{N} = 6$  superconformal Chern–Simons–matter theory and will find that in the BMN scaling limit the two-loop  $\mathcal{N} = 6$  theory resembles the one loop  $\mathcal{N} = 4$  SYM theory. Away from the scaling limit the  $\mathcal{N} = 6$  dilatation generator has additional terms. The mixing problem of the BMN limit of  $\mathcal{N} = 4$  SYM was never solved beyond the planar limit even perturbatively in  $\frac{1}{N}$  due to complications arising from huge degeneracies in the planar spectrum [28]. A third type of operators one could dream of studying beyond the planar limit are operators dual to spinning strings. Such operators typically contain  $M$  excitations and  $J$  background fields where  $J, M \rightarrow \infty$  with  $\frac{M}{J}$  finite. For such operators, however, acting with the dilatation generator involves evaluating infinitely many terms and writing down the dilatation generator exactly seems intractable. In reference [29] it was suggested that non-planar corrections to operators dual to spinning strings could be treated using a coherent state formalism.

Non-planar effects in  $\mathcal{N} = 6$  superconformal Chern–Simons–matter theory should reflect interactions in the dual type IIA string theory. Directly comparable quantities are, however, not immediate to write down, not least because the  $AdS_4/CFT_3$  duality implies the following relation between the string coupling constant and the gauge theory parameters [9]

$$g_s = \frac{\lambda^{5/4}}{N}. \quad (1)$$

This should be compared to the similar relation for  $\mathcal{N} = 4$  SYM that took the form  $g_s = \frac{\lambda}{N}$  which at least gave the hope that interacting BMN string states could be studied by perturbative gauge theory computations. The comparison between the perturbative non-planar gauge theory and the interacting string theory, described in terms of light cone string field theory on a plane wave, however, remained inconclusive. For a recent review, see [30]. It is thus primarily with the purpose of investigating the role of integrability beyond the planar limit and the structural similarities and differences between  $\mathcal{N} = 4$  SYM and  $\mathcal{N} = 6$  superconformal Chern–Simons–matter theory that we engage into the present investigations.

We start in section 2 by giving an ultra-short summary of  $\mathcal{N} = 6$  superconformal Chern–Simons–matter theory, i.e. ABJM theory. Subsequently in section 3 we derive the full two-loop dilatation generator in the  $SU(2) \times SU(2)$  sector, deferring the details to Appendix A. After a short discussion of the structure of the dilatation generator in section 4 we explain in section 5 the relation between planar degeneracies and conserved charges. Then we proceed to apply the dilatation generator to respectively short operators in section 6 and BMN operators in section 7. Finally, section 8 contains our conclusion.

## 2 ABJM theory

Our notation will follow that of references [31, 11]. ABJM theory is a three-dimensional superconformal Chern–Simons–matter theory with gauge group  $U(N)_k \times U(N)_{-k}$  and  $R$ -symmetry group  $SU(4)$ . The parameter  $k$  denotes the Chern–Simons level. The fields of ABJM theory consist of gauge fields  $A_m$  and  $\bar{A}_m$ , complex scalars  $Y^I$  and Majorana spinors  $\Psi_I$ ,  $I \in \{1, \dots, 4\}$ . The two gauge fields belong to the adjoint representation of the two  $U(N)$ 's. The scalars  $Y^I$  and the spinors  $\Psi_I$  transform in the  $N \times \bar{N}$  representa-

tion of the gauge group and in the fundamental and anti-fundamental representation of  $SU(4)$  respectively. For our purposes it proves convenient to write the scalars and spinors explicitly in terms of their  $SU(2)$  component fields, i.e. [31]

$$\begin{aligned} Y^I &= \{Z^A, W^{\dagger A}\}, & Y_I^\dagger &= \{Z_A^\dagger, W_A\}, \\ \Psi_I &= \{\epsilon_{AB} \xi^B e^{i\pi/4}, \epsilon_{AB} \omega^{\dagger B} e^{-i\pi/4}\}, \\ \Psi^{I\dagger} &= \{-\epsilon^{AB} \xi_B^\dagger e^{-i\pi/4}, -\epsilon^{AB} \omega_B e^{i\pi/4}\}, \end{aligned}$$

where now  $A, B \in \{1, 2\}$ . Expressed in terms of these fields the action reads

$$\begin{aligned} S &= \int d^3x \left[ \frac{k}{4\pi} \epsilon^{mnp} \text{Tr}(A_m \partial_n A_p + \frac{2i}{3} A_m A_n A_p) - \frac{k}{4\pi} \epsilon^{mnp} \text{Tr}(\bar{A}_m \partial_n \bar{A}_p + \frac{2i}{3} \bar{A}_m \bar{A}_n \bar{A}_p) \right. \\ &\quad \left. - \text{Tr}(\mathcal{D}_m Z)^\dagger \mathcal{D}^m Z - \text{Tr}(\mathcal{D}_m W)^\dagger \mathcal{D}^m W + i \text{Tr} \xi^\dagger \not{D} \xi + i \text{Tr} \omega^\dagger \not{D} \omega - V^{ferm} - V^{bos} \right]. \end{aligned}$$

Here the covariant derivatives are defined as

$$\mathcal{D}_m Z^A = \partial_m Z^A + i A_m Z^A - i Z^A \bar{A}_m, \quad \mathcal{D}_m W_A = \partial_m W_A + i \bar{A}_m W_A - i W_A A_m, \quad (2)$$

and similarly for  $\mathcal{D}_m \xi^B$  and  $\mathcal{D}_m \omega_B$ . The bosonic as well as the fermionic potential can be separated into D-terms and F-terms which read

$$\begin{aligned} V_D^{ferm} &= \frac{2\pi i}{k} \text{Tr} \left[ (\xi^A \xi_A^\dagger - \omega^{\dagger A} \omega_A) (Z^B Z_B^\dagger - W^{\dagger B} W_B) - (\xi_A^\dagger \xi^A - \omega_A \omega^{\dagger A}) (Z_B^\dagger Z^B - W_B W^{\dagger B}) \right] \\ &\quad + \frac{4\pi i}{k} \text{Tr} \left[ (\xi^A Z_A^\dagger - W^{\dagger A} \omega_A) (Z^B \xi_B^\dagger - \omega^{\dagger B} W_B) - (Z_A^\dagger \xi^A - \omega_A W^{\dagger A}) (\xi_B^\dagger Z^B - W_B \omega^{\dagger B}) \right], \\ V_F^{ferm} &= \frac{2\pi}{k} \epsilon_{AC} \epsilon^{BD} \text{Tr} \left[ 2\xi^A W_B Z^C \omega_D + 2\xi^A \omega_B Z^C W_D + Z^A \omega_B Z^C \omega_D + \xi^A W_B \xi^C W_D \right] \\ &\quad + \frac{2\pi}{k} \epsilon^{AC} \epsilon_{BD} \text{Tr} \left[ 2\xi_A^\dagger W^{\dagger B} Z_C^\dagger \omega^{\dagger D} + 2\xi_A^\dagger \omega^{\dagger B} Z_C^\dagger W^{\dagger D} + Z_A^\dagger \omega^{\dagger B} Z_C^\dagger \omega^{\dagger D} + \xi_A^\dagger W^{\dagger B} \xi_C^\dagger W^{\dagger D} \right], \\ V_D^{bos} &= \left( \frac{2\pi}{k} \right)^2 \text{Tr} \left[ (Z^A Z_A^\dagger + W^{\dagger A} W_A) (Z^B Z_B^\dagger - W^{\dagger B} W_B) (Z^C Z_C^\dagger - W^{\dagger C} W_C) \right. \\ &\quad + (Z_A^\dagger Z^A + W_A W^{\dagger A}) (Z_B^\dagger Z^B - W_B W^{\dagger B}) (Z_C^\dagger Z^C - W_C W^{\dagger C}) \\ &\quad - 2Z_A^\dagger (Z^B Z_B^\dagger - W^{\dagger B} W_B) Z^A (Z_C^\dagger Z^C - W_C W^{\dagger C}) \\ &\quad \left. - 2W^{\dagger A} (Z_B^\dagger Z^B - W_B W^{\dagger B}) W_A (Z_C^\dagger Z^C - W_C W^{\dagger C}) \right] \quad (3) \end{aligned}$$

and

$$\begin{aligned} V_F^{bos} &= - \left( \frac{4\pi}{k} \right)^2 \text{Tr} \left[ W^{\dagger A} Z_B^\dagger W^{\dagger C} W_A Z^B W_C - W^{\dagger A} Z_B^\dagger W^{\dagger C} W_C Z^B W_A \right. \\ &\quad \left. + Z_A^\dagger W^{\dagger B} Z_C^\dagger Z^A W_B Z^C - Z_A^\dagger W^{\dagger B} Z_C^\dagger Z^C W_B Z^A \right]. \quad (4) \end{aligned}$$

Introducing a 't Hooft parameter for the theory

$$\lambda = \frac{4\pi N}{k}, \quad (5)$$

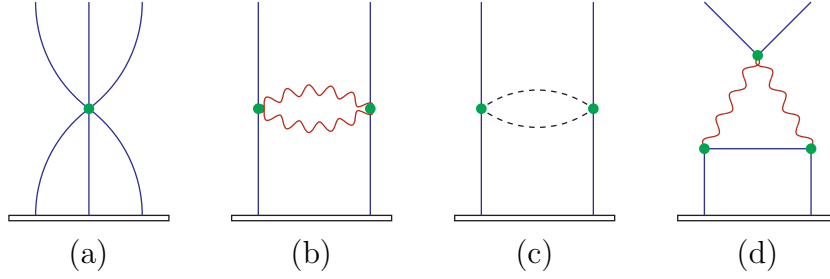


Figure 1: The four types of two-loop diagrams contributing to anomalous dimensions. For operators in the  $SU(2) \times SU(2)$  sector diagrams in class (d) do not contribute.

one can consider the 't Hooft limit

$$N \rightarrow \infty, \quad k \rightarrow \infty, \quad \lambda \text{ fixed.} \quad (6)$$

Furthermore, the theory has a double expansion in  $\lambda$  and  $\frac{1}{N}$ . In this paper we will be interested in studying non-planar effects for anomalous dimensions at the leading two-loop level.

### 3 The derivation of the full dilatation generator

In [10, 11] an expression for the planar dilatation generator acting on operators of the type

$$\mathcal{O} = \text{Tr}(Y^{A_1} Y_{B_1}^\dagger Y^{A_2} Y_{B_2}^\dagger \dots Y^{A_L} Y_{B_L}^\dagger), \quad (7)$$

where  $A_i, B_i \in \{1, 2\}$  was derived and proved to be identical to the Hamiltonian of an integrable alternating  $SU(4)$  spin chain.

Here we will restrict ourselves to considering scalar operators belonging to a  $SU(2) \times SU(2)$  sub-sector i.e. operators of the following type

$$\mathcal{O} = \text{Tr} \left( Z^{A_1} W_{B_1} \dots Z^{A_L} W_{B_L} \right), \quad (8)$$

and their multi-trace generalizations. For this class of operators we wish to derive the full dilatation generator including non-planar contributions. In order to do so we employ the method of effective vertices from reference [32]. An effective vertex is a vertex which encodes the combinatorics of a given type of Feynman diagram. For instance, the scalar D-terms give rise to the following effective vertex contributing to the dilatation generator

acting on operators of the type given in eqn. (8)

$$\begin{aligned}
(V_D^{bos})^{eff} = \gamma : \text{Tr} & \left[ \left( Z^A Z_A^\dagger + W^{\dagger A} W_A \right) \left( Z^B Z_B^\dagger - W^{\dagger B} W_B \right) \left( Z^C Z_C^\dagger - W^{\dagger C} W_C \right) \right. \\
& + \left( Z_A^\dagger Z^A + W_A W^{\dagger A} \right) \left( Z_B^\dagger Z^B - W_B W^{\dagger B} \right) \left( Z_C^\dagger Z^C - W_C W^{\dagger C} \right) \\
& - 2 Z_A^\dagger \left( Z^B Z_B^\dagger - W^{\dagger B} W_B \right) Z^A \left( Z_C^\dagger Z^C - W_C W^{\dagger C} \right) \\
& \left. - 2 W^{\dagger A} \left( Z_B^\dagger Z^B - W_B W^{\dagger B} \right) W_A \left( Z_C^\dagger Z^C - W^{\dagger C} W_C \right) \right] :
\end{aligned} \tag{9}$$

where each daggered field is supposed to be contracted with a field inside  $\mathcal{O}$ , the omissions of self-contractions of the vertex being encoded in the symbol  $::$ . All contractions of  $(V_D^{bos})^{eff}$  with the operator  $\mathcal{O}$  multiply the same Feynman integral whose value we denote as  $\gamma$ . The relevant integral is represented by the Feynman diagram in Fig 1a. The dilatation generator also gets contributions from the bosonic  $F$ -terms, gluon exchange (Fig. 1b), fermion exchange (Fig. 1c) and scalar self interactions [10, 11]. Notice, however, that for operators belonging to the  $SU(2) \times SU(2)$  sector there are no contributions involving paramagnetic interactions (Fig. 1d). If things work as in  $\mathcal{N} = 4$  SYM the contribution from the D-terms in the sixth order scalar potential should cancel against contributions from gluon exchange, fermion exchange and self-interactions to all orders in the genus expansion. We show explicitly in Appendix A that this is indeed the case. We thus have that the full two-loop dilatation generator takes the form

$$D =: V_F^{bos} : \tag{10}$$

It is easy to see that the dilatation generator vanishes when acting on an operator consisting of only two of the four fields from the  $SU(2) \times SU(2)$  sector. Accordingly we will denote two of the fields, say  $Z_1$  and  $W_1$ , as background fields and  $Z_2$  and  $W_2$  as excitations. It is likewise easy to see that operators with only one type of excitations, say  $W_2$ 's, form a closed set under dilatations. For operators with only  $W_2$ -excitations the dilatation generator takes the form

$$\begin{aligned}
D = - \left( \frac{4\pi}{k} \right)^2 : \text{Tr} & \left[ W^{\dagger 2} Z_1^\dagger W^{\dagger 1} W_2 Z^1 W_1 - W^{\dagger 2} Z_1^\dagger W^{\dagger 1} W_1 Z^1 W_2 \right. \\
& \left. + W^{\dagger 1} Z_1^\dagger W^{\dagger 2} W_1 Z^1 W_2 - W^{\dagger 1} Z_1^\dagger W^{\dagger 2} W_2 Z^1 W_1 \right] :
\end{aligned} \tag{11}$$

In the case of two different types of excitations, i.e. both  $W_2$ 's and  $Z_2$ 's, the dilatation generator has 16 terms. It appears from the one in (11) by adding similar terms with 1 and 2 interchanged and subsequently adding the same operator with  $Z$  and  $W$  interchanged. In both cases  $D$  is easily seen to reduce to the one of [10, 11] in the planar limit

$$D_{planar} \equiv \lambda^2 D_0 = \lambda^2 \sum_{k=1}^{2L} (1 - P_{k,k+2}), \tag{12}$$

where  $P_{k,k+2}$  denotes the permutation between sites  $k$  and  $k+2$  and  $2L$  denotes the total number of fields inside an operator. As explained in [10, 11] this is the Hamiltonian of two Heisenberg magnets living respectively on the odd and the even sites of a spin chain. The two magnets are coupled via the constraint that the total momentum of their excitations should vanish which is needed to ensure the cyclicity of the trace.

## 4 The structure of the dilatation generator

As proved in the previous section and in Appendix A the two-loop dilatation generator in the  $SU(2) \times SU(2)$  sector takes the form given in eqn. (10). When acting on a given operator we have to perform three contractions as dictated by the three hermitian conjugate fields. It is easy to see that by acting with the dilatation generator one can change the number of traces in a given operator by at most two.<sup>2</sup> More precisely, the two loop dilatation generator has the expansion

$$D = \lambda^2 \left( D_0 + \frac{1}{N} D_+ + \frac{1}{N} D_- + \frac{1}{N^2} D_{00} + \frac{1}{N^2} D_{++} + \frac{1}{N^2} D_{--} \right). \quad (13)$$

Here  $D_+$  and  $D_{++}$  increase the number of traces by one and two respectively and  $D_-$  and  $D_{--}$  decrease the number of traces by one and two. Finally,  $D_0$  and  $D_{00}$  do not change the number of traces. We notice that in  $\mathcal{N} = 4$  SYM the two-loop dilatation generator in the  $SU(2)$  sector has a similar expansion [2] whereas the most studied, one-loop dilatation generator involves only two contractions and does not contain any  $\frac{1}{N^2}$  terms [33, 34, 32, 35]. Let us assume that we have found an eigenstate of the planar dilatation generator  $D_0$ , i.e.

$$D_0|\mathcal{O}\rangle = E_{\mathcal{O}}|\mathcal{O}\rangle, \quad (14)$$

and let us treat the terms sub-leading in  $\frac{1}{N}$  as a perturbation. First, let us assume that there are no degeneracies between  $n$ -trace states and  $(n+1)$ -trace states in the spectrum or that the perturbation has no matrix elements between such degenerate states. If that is the case we can proceed by using non-degenerate quantum mechanical perturbation theory. Clearly, the leading  $\frac{1}{N}$  terms do not have any diagonal components so the energy correction for the state  $|\mathcal{O}\rangle$  reads:

$$\delta E_{\mathcal{O}} = \frac{\lambda^2}{N^2} \sum_{\mathcal{K} \neq \mathcal{O}} \frac{\langle \mathcal{O} | D_+ + D_- | \mathcal{K} \rangle \langle \mathcal{K} | D_+ + D_- | \mathcal{O} \rangle}{E_{\mathcal{O}} - E_{\mathcal{K}}} + \frac{\lambda^2}{N^2} \langle \mathcal{O} | D_{00} | \mathcal{O} \rangle. \quad (15)$$

If there are degeneracies between  $n$ -trace states and  $(n+1)$ -trace states we have to diagonalize the perturbation in the subset of degenerate states and the corrections will typically be of order  $\frac{1}{N}$ .

## 5 Planar parity pairs, conserved charges and integrability

In the previous sections we derived the two-loop non-planar dilatation generator for the  $SU(2) \times SU(2)$  sector and analyzed its structure. From the work of [10, 11] we know that the planar part of the dilatation generator can be identified as the Hamiltonian for an integrable  $SU(2) \times SU(2)$  spin chain. It is then interesting to ask what happens

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<sup>2</sup>Acting with the dilatation generator involves performing three contractions. Performing the first of these does not change the number of traces. Each of the subsequent contractions on the other hand can lead to an increase or decrease of the trace number by one.

to integrability once non-planar corrections are taken into account. One approach to answering this question is to consider *planar parity pairs*, as we will now review.

As part of their analysis of the dilatation generator of  $\mathcal{N} = 4$  SYM, the authors of [2] considered its action on short scalar operators. They observed an a priori unexpected degeneracy in the resulting spectra, between operators with the same trace structure but opposite *parity*, where the latter is defined as the operation that reverses the order of all generators within each trace (in other words, complex conjugation of the gauge group generators) [36]. Parity commutes with the action of the dilatation generator (and is thus a conserved quantity), therefore one expects that the various operators will organize themselves into distinct sectors according to their (positive or negative) parity. Positive and negative parity sectors do not mix with each other and there is no reason to expect any relation between their spectra.

However, in [2] it was observed that every time there exist operators, which have the same trace structure and belong to the same global  $SO(6)$  representation but have opposite parity, their *planar* anomalous dimensions turn out to be equal. This degeneracy could be very simply understood as a consequence of parity symmetry and planar integrability: Recall that one of the hallmarks of integrability is the existence of a tower of commuting conserved charges  $Q_n$  (the hamiltonian  $Q_2$  being just one of them). For the  $\mathcal{N} = 4$  SYM spin chain there exists such a charge  $Q_3$  which (being conserved) commutes with the dilatation generator but *anticommutes* with the operation of parity. This clearly implies the existence of pairs of operators with opposite parity and equal anomalous dimension at the planar level. Thus planar integrability manifests itself in the spectrum of short operators through the appearance of degeneracies between planar parity pairs. Moving beyond planar level, it was observed in [2] that all these degeneracies are lifted: There is no apparent relation between the different parity sectors in the spectrum of the non-planar dilatation generator. This was taken as an indication (though by no means a proof) that integrability is lost once one considers non-planar corrections. In this connection, it is worth noticing that the degeneracies observed at planar one-loop order remain when planar higher loop corrections are taken into account [2].

Returning to  $\mathcal{N} = 6$  ABJM theory, it is interesting to ask whether the same pattern of planar degeneracies which are lifted at the non-planar level arises in the present context. We begin by defining a parity operation which inverts the order of all generators within each trace, for example:

$$\text{Tr} [Z_1 W_1 Z_1 W_2 Z_2 W_1] \longrightarrow \text{Tr} [W_1 Z_2 W_2 Z_1 W_1 Z_1] = \text{Tr} [Z_1 W_1 Z_1 W_1 Z_2 W_2]. \quad (16)$$

Obviously, the Hamiltonian of the  $SU(2) \times SU(2)$  spin chain is parity symmetric. Furthermore, from the work of [10, 11] we know that the conserved charges of the  $SU(2) \times SU(2)$  spin chain are nothing but the sum of the charges of the two  $SU(2)$  Heisenberg spin chains. In particular, the third charge  $Q_3$  again anti-commutes with parity while commuting with the Hamiltonian.

Hence we conclude that we should expect to see parity pairs in the planar part of the spectrum. Furthermore, the intuition gained from  $\mathcal{N} = 4$  SYM points to these degeneracies being broken once non-planar corrections are taken into account. In the following section, by explicitly considering the action of the dilatation generator on a series of short operators, we will see that both these expectations are confirmed.



## 6 Short Operators

In this section we determine non-planar corrections to a number of short operators. This is done by explicitly computing and diagonalizing the mixing matrix (aided by `GPL Maxima` as well as `Mathematica`).

### 6.1 Operators with only one type of excitation

Operators with only one type of excitation can, at the planar level, be described in terms of just a single Heisenberg spin chain and behave at the leading two-loop level very similarly to their  $\mathcal{N} = 4$  SYM cousins at one-loop level. Notice, however, that once one goes beyond the planar limit the dilatation generator has novel  $\frac{1}{N^2}$  terms. The simplest set of operators for which one observes degenerate parity pairs as well as non-trivial mixing between operators with different number of traces consists of operators of length 14 with three excitations. There are in total 17 such non-protected operators. Notice that due to the absence of the trace condition of  $\mathcal{N} = 4$  SYM, for which the gauge group is  $SU(N)$ , there are more operators here than the naive generalizations of the  $\mathcal{N} = 4$  SYM ones. Among the non-protected operators there are only 8 which are not descendants and which we list below. (To improve readability we suppress the background  $Z_1$  fields.) Notice that only  $\mathcal{O}_1$ ,  $\mathcal{O}_3$  and  $\mathcal{O}_6$  have analogues in  $\mathcal{N} = 4$  SYM.

$$\begin{aligned}
\mathcal{O}_1 &= \text{Tr}([W_1 W_1, W_1 W_2] W_1 W_2 W_2) \\
\mathcal{O}_2 &= \text{Tr}(W_1) \text{Tr}(W_1 [W_1, W_2] W_1 W_2 W_2) \\
\mathcal{O}_3 &= 2\text{Tr}(W_1 W_1 W_1 W_1 W_2 W_2 W_2) - 3\text{Tr}(W_1 W_2 W_2 W_1 W_1 W_1 W_2) \\
&\quad - 3\text{Tr}(W_1 W_2 W_1 W_1 W_1 W_2 W_2) + 2\text{Tr}(W_1 W_2 W_1 W_2 W_1 W_1 W_2) \\
&\quad + 2\text{Tr}(W_1 W_1 W_2 W_1 W_1 W_2 W_2) \\
\mathcal{O}_4 &= 4(2 + \sqrt{5})\text{Tr}(W_2) \text{Tr}(W_1 W_1 W_1 W_2 W_1 W_2) - 2(1 + \sqrt{5})\text{Tr}(W_2) \text{Tr}(W_1 W_1 W_1 W_1 W_2 W_2) \\
&\quad - 2(3 + \sqrt{5})\text{Tr}(W_2) \text{Tr}(W_1 W_1 W_2 W_1 W_1 W_2) + (3 + \sqrt{5})\text{Tr}(W_1) \text{Tr}(W_1 W_1 W_2 W_1 W_2 W_2) \\
&\quad + (3 + \sqrt{5})\text{Tr}(W_1) \text{Tr}(W_1 W_2 W_1 W_1 W_2 W_2) - 2\text{Tr}(W_1) \text{Tr}(W_1 W_1 W_1 W_2 W_2 W_2) \\
&\quad - 2(2 + \sqrt{5})\text{Tr}(W_1) (W_2 W_1 W_2 W_1 W_2 W_1) \\
\mathcal{O}_5 &= -4(2 - \sqrt{5})\text{Tr}(W_2) \text{Tr}(W_1 W_1 W_1 W_2 W_1 W_2) + 2(1 - \sqrt{5})\text{Tr}(W_2) \text{Tr}(W_1 W_1 W_1 W_1 W_2 W_2) \\
&\quad + 2(3 - \sqrt{5})\text{Tr}(W_2) \text{Tr}(W_1 W_1 W_2 W_1 W_1 W_2) - (3 - \sqrt{5})\text{Tr}(W_1) \text{Tr}(W_1 W_1 W_2 W_1 W_2 W_2) \\
&\quad - (3 - \sqrt{5})\text{Tr}(W_1) \text{Tr}(W_1 W_2 W_1 W_1 W_2 W_2) + 2\text{Tr}(W_1) \text{Tr}(W_1 W_1 W_1 W_2 W_2 W_2) \\
&\quad + 2(2 - \sqrt{5})\text{Tr}(W_1) \text{Tr}(W_2 W_1 W_2 W_1 W_2 W_1) \\
\mathcal{O}_6 &= \text{Tr}(W_1 W_1) \text{Tr}(W_1 [W_2, W_1] W_2 W_2) + \text{Tr}(W_1 W_2) \text{Tr}(W_1 W_1 [W_1, W_2] W_2) \\
\mathcal{O}_7 &= \text{Tr}(W_1) \text{Tr}(W_1) \text{Tr}(W_1 [W_2, W_1] W_2 W_2) + \text{Tr}(W_2) \text{Tr}(W_1) \text{Tr}(W_1 W_1 [W_1, W_2] W_2) \\
\mathcal{O}_8 &= \text{Tr}(W_2) \text{Tr}(W_1 W_1) \text{Tr}(W_1 [W_2, W_1] W_2) + \text{Tr}(W_1) \text{Tr}(W_1 W_2) \text{Tr}(W_1 [W_1, W_2] W_2)
\end{aligned} \tag{17}$$

The associated planar anomalous dimensions (in units of  $\lambda^2$ ), trace structure and parity are

Eigenvector	Eigenvalue	Trace structure	Parity
$\mathcal{O}_1$	5	(14)	−
$\mathcal{O}_2$	6	(2)(12)	−
$\mathcal{O}_3$	5	(14)	+
$\mathcal{O}_4$	$5 + \sqrt{5}$	(2)(12)	+
$\mathcal{O}_5$	$5 - \sqrt{5}$	(2)(12)	+
$\mathcal{O}_6$	4	(4)(10)	+
$\mathcal{O}_7$	4	(2)(2)(10)	+
$\mathcal{O}_8$	6	(2)(4)(8)	+

where by parity for multi-trace operators we mean the product of the parity of its single trace components. The planar anomalous dimensions of  $\mathcal{O}_1$ ,  $\mathcal{O}_3$  and  $\mathcal{O}_6$  agree (as they should) with those of the similar operators in  $\mathcal{N} = 4$  SYM, cf. [2]. We have one pair of degenerate single trace operators with opposite parity, namely the operators  $\mathcal{O}_1$  and  $\mathcal{O}_3$ .<sup>3</sup>

Expressing the dilatation generator in the basis above and taking into account all non-planar corrections we get

$$\begin{pmatrix} 5 + \frac{15}{N^2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{6}{N^2} & 6 + \frac{24}{N^2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 + \frac{35}{N^2} & 0 & 0 & -\frac{8}{N} & -\frac{4}{N^2} & -\frac{2}{N^2} \\ 0 & 0 & -\frac{\sqrt{5}}{N} & 5 + \sqrt{5} + \frac{(5\sqrt{5}+35)}{N^2} & \frac{3\sqrt{5}-5}{N^2} & \frac{1}{N^2} & 0 & \frac{2}{N} \\ 0 & 0 & -\frac{\sqrt{5}}{N} & -\frac{5+3\sqrt{5}}{N^2} & 5 - \sqrt{5} - \frac{5\sqrt{5}-35}{N^2} & -\frac{1}{N^2} & 0 & -\frac{2}{N} \\ 0 & 0 & -\frac{20}{N} & \frac{4\sqrt{5}+20}{N^2} & -\frac{20-4\sqrt{5}}{N^2} & 4 + \frac{28}{N^2} & 0 & 0 \\ 0 & 0 & -\frac{10}{N^2} & \frac{4\sqrt{5}+20}{N} & \frac{4\sqrt{5}-20}{N} & 0 & 4 + \frac{32}{N^2} & -\frac{2}{N^2} \\ 0 & 0 & -\frac{10}{N^2} & \frac{24\sqrt{5}+40}{N} & \frac{24\sqrt{5}-40}{N} & \frac{8}{N} & -\frac{8}{N^2} & 6 + \frac{40}{N^2} \end{pmatrix}. \quad (18)$$

Notice the decoupling of positive and negative parity states and the presence of numerous  $\frac{1}{N^2}$ -terms which do not have analogues in one-loop  $\mathcal{N} = 4$  SYM. One observes that the states  $\mathcal{O}_1$  and  $\mathcal{O}_2$  are exact eigenstates of the full dilatation generator with non-planar corrections equal to

$$\delta E_1 = \frac{15}{N^2}, \quad \delta E_2 = \frac{24}{N^2}. \quad (19)$$

For the remaining operators we observe that all matrix elements between degenerate states vanish. Thus the leading non-planar corrections to the anomalous dimensions can be found using second order non-degenerate perturbation theory. The results read

$$\begin{aligned} \delta E_3 &= \frac{195}{N^2}, & \delta E_4 &= \frac{115 + 37\sqrt{5}}{N^2}, \\ \delta E_5 &= \frac{115 - 37\sqrt{5}}{N^2}, & \delta E_6 &= -\frac{132}{N^2}, \\ \delta E_7 &= \frac{32}{N^2}, & \delta E_8 &= -\frac{120}{N^2}. \end{aligned} \quad (20)$$

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<sup>3</sup>We also observe a degeneracy between the negative parity double trace state  $\mathcal{O}_2$  and the positive parity triple trace state  $\mathcal{O}_8$  as well as a degeneracy between the double trace state  $\mathcal{O}_6$  and the triple trace state  $\mathcal{O}_7$  both of positive parity. However, states with different numbers of traces can not be connected via the conserved charge  $Q_3$ .

We observe that all degeneracies found at the planar level get lifted when non-planar corrections are taken into account. This in particular holds for the degeneracies between the members of the planar parity pair  $(\mathcal{O}_1, \mathcal{O}_3)$ . Notice that whereas the planar eigenvalues of the operators  $\mathcal{O}_1$ ,  $\mathcal{O}_3$  and  $\mathcal{O}_6$  are identical to those of their  $\mathcal{N} = 4$  SYM cousins the non-planar corrections are not.

## 6.2 Operators with two types of excitations

An operator with two excitations of different type corresponds in spin chain language to the situation where each of the two coupled spin chains has one excitation. Such an operator does not immediately have an analogue in  $\mathcal{N} = 4$  SYM. (One can indeed consider scalar  $\mathcal{N} = 4$  SYM operators with two types of excitations  $\Phi$  and  $\Psi$  on a background of  $Z$  fields but these operators should be organized into representations of  $SO(6)$ , and not of  $SU(2) \times SU(2)$  as here, and thus always come in symmetrized or antisymmetrized versions.)

### 6.2.1 Length 8 with 2 excitations

Let us analyze the simplest multiplet of operators with two excitations of different types that exhibit some of the above mentioned non-trivial features of the  $\frac{1}{N}$ -expansion, operators of length eight with one excitation of each type. There are in total 7 such non-protected operators. The planar non-protected eigenstates of the two-loop dilatation generator read

$$\begin{aligned}
\mathcal{O}_1 &= \text{Tr}(Z_1 W_1 \{Z_1 W_2, Z_2 W_1\} Z_1 W_1) - \text{Tr}(W_1 Z_1 \{W_1 Z_2, W_2 Z_1\} W_1 Z_1) \\
\mathcal{O}_2 &= -\text{Tr}(Z_1 W_1 [Z_1 W_2, Z_2 W_1] Z_1 W_1) + \text{Tr}(W_1 Z_1 [W_1 Z_2, W_2 Z_1] W_1 Z_1) \\
\mathcal{O}_3 &= \text{Tr}(Z_1 W_1 Z_1 W_1) [\text{Tr}(Z_1 W_2 Z_2 W_1) - \text{Tr}(W_1 Z_2 W_2 Z_1)] \\
\mathcal{O}_4 &= \text{Tr}(W_1 Z_1) [\text{Tr}(W_1 Z_1 W_2 Z_2 W_1 Z_1) - \text{Tr}(Z_1 W_1 Z_2 W_2 Z_1 W_1)] \\
\mathcal{O}_5 &= \text{Tr}(W_1 Z_1) \text{Tr}(W_1 Z_1) [\text{Tr}(W_2 Z_1 W_1 Z_2) - \text{Tr}(Z_2 W_1 Z_1 W_2)] \\
\mathcal{O}_6 &= -\text{Tr}(Z_1 W_1 [Z_1 W_2, Z_2 W_1] Z_1 W_1) - \text{Tr}(W_1 Z_1 [W_1 Z_2, W_2 Z_1] W_1 Z_1) \\
\mathcal{O}_7 &= -\text{Tr}(W_1 Z_1) [\text{Tr}(W_1 Z_1 [W_1 Z_2, W_2 Z_1]) + \text{Tr}(Z_1 W_1 [Z_1 W_2, Z_2 W_1])]
\end{aligned} \tag{21}$$

and the associated planar anomalous dimensions (in units of  $\lambda^2$ ), trace structure and parity are

Eigenvector	Eigenvalue	Trace Structure	Parity
$\mathcal{O}_1$	8	(8)	—
$\mathcal{O}_2$	4	(8)	—
$\mathcal{O}_3$	8	(4)(4)	—
$\mathcal{O}_4$	6	(2)(6)	—
$\mathcal{O}_5$	8	(2)(2)(4)	—
$\mathcal{O}_6$	4	(8)	+
$\mathcal{O}_7$	6	(2)(6)	+

Notice that we have two pairs of degenerate operators with opposite parity, namely the single trace operators  $\mathcal{O}_2$  and  $\mathcal{O}_6$  and the double trace operators  $\mathcal{O}_4$  and  $\mathcal{O}_7$ .<sup>4</sup>

Expressing the dilatation generator in the basis given above and taking into account all non-planar corrections we get

$$\begin{pmatrix} 8 & \frac{8}{N^2} & \frac{16}{N} & \frac{4}{N} & -\frac{8}{N^2} & 0 & 0 \\ \frac{8}{N^2} & 4 - \frac{12}{N^2} & 0 & -\frac{2}{N} & -\frac{4}{N^2} & 0 & 0 \\ \frac{16}{N} & -\frac{8}{N} & 8 & 0 & 0 & 0 & 0 \\ 0 & -\frac{16}{N} & -\frac{8}{N^2} & 6 - \frac{8}{N^2} & -\frac{12}{N} & 0 & 0 \\ 0 & \frac{8}{N^2} & 0 & -\frac{12}{N} & 8 - \frac{8}{N^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 + \frac{4}{N^2} & \frac{2}{N} \\ 0 & 0 & 0 & 0 & 0 & \frac{8}{N} & 6 + \frac{8}{N^2} \end{pmatrix}. \quad (22)$$

The non-planar corrections for  $\mathcal{O}_6$  and  $\mathcal{O}_7$  can be found exactly and read

$$\delta E_{6,7} = \frac{6}{N^2} \mp \left( \sqrt{1 + \frac{20}{N^2} + \frac{4}{N^4}} - 1 \right). \quad (23)$$

The corrections to the eigenvalues of the remaining operators we instead find using perturbation theory as described in section 4. First we notice that most matrix elements between degenerate states vanish. The only exception are the matrix elements between the states  $\mathcal{O}_1$  and  $\mathcal{O}_3$ . To find the non-planar correction to the energy of these states we diagonalize the Hamiltonian in the corresponding subspace and find

$$\delta E_{1,3} = \mp \frac{16}{N}. \quad (24)$$

For the remaining operators the leading non-planar corrections to the energy can be found using second order non-degenerate perturbation theory. The results read

$$\delta E_2 = -\frac{28}{N^2}, \quad \delta E_4 = -\frac{64}{N^2}, \quad \delta E_5 = \frac{64}{N^2}. \quad (25)$$

We again notice that all degeneracies observed at the planar level get lifted when non-planar corrections are taken into account. This in particular holds for the degeneracies between the members of the two parity pairs.

### 6.2.2 Length 8 with 3 excitations

We now consider operators with three excitations, one of type  $Z_2$  and two of type  $W_2$ . Among this type of operators one finds 7 which are descendants of the 7 operators considered in the previous section. Of highest weight states one has the following four planar

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<sup>4</sup>The double trace operators  $\mathcal{O}_4$  and  $\mathcal{O}_7$  can be related via  $Q_3$  when letting  $Q_3$  act only on the longer of the two constituent traces of the operators.

eigenstates:

$$\begin{aligned}
\mathcal{O}_1 &= \text{Tr}(Z_1 W_2) [\text{Tr}(Z_1 W_1 Z_2 W_2 Z_1 W_1) - \text{Tr}(W_1 Z_1 W_2 Z_2 W_1 Z_1)] \\
&\quad - \text{Tr}(Z_1 W_1) [\text{Tr}(Z_1 W_1 Z_2 W_2 Z_1 W_2) - \text{Tr}(Z_1 W_2 Z_2 W_1 Z_1 W_2)] \\
\mathcal{O}_2 &= \text{Tr}(Z_1 W_1 [Z_2 W_1, Z_1 W_2] Z_1 W_2) + \text{Tr}(Z_1 W_2 [Z_1 W_2, Z_2 W_1] Z_1 W_1) \\
&\quad + \text{Tr}(Z_1 W_1 [Z_1 W_1, Z_2 W_2] Z_1 W_2) + \text{Tr}(Z_1 W_2 [Z_2 W_2, Z_1 W_1] Z_1 W_1) \\
\mathcal{O}_3 &= -\text{Tr}(W_2 Z_1 [W_1 Z_1, W_1 Z_2] W_2 Z_1) + \text{Tr}(W_1 Z_1 [W_2 Z_2, W_2 Z_1] W_1 Z_1) \\
\mathcal{O}_4 &= \text{Tr}(Z_1 W_2) [\text{Tr}(W_1 Z_1 [W_1 Z_2, W_2 Z_1]) + \text{Tr}(Z_1 W_1 [Z_1 W_2, Z_2 W_1])] \\
&\quad + \text{Tr}(Z_1 W_1) [\text{Tr}(Z_1 W_2 [Z_1 W_1, Z_2 W_2]) + \text{Tr}(W_2 Z_1 [W_2 Z_2, W_1 Z_1])]
\end{aligned} \tag{26}$$

Their planar anomalous dimensions (in units of  $\lambda^2$ ), trace structure and parity are tabulated below.

Eigenvector	Eigenvalue	Trace Structure	Parity
$\mathcal{O}_1$	6	(2)(6)	−
$\mathcal{O}_2$	6	(8)	+
$\mathcal{O}_3$	6	(8)	+
$\mathcal{O}_4$	6	(2)(6)	+

We observe one planar parity pair with trace structure (2)(6). The full mixing matrix for this set of states takes the following form:

$$\begin{pmatrix} 6 - \frac{16}{N^2} & 0 & 0 & 0 \\ 0 & 6 + \frac{12}{N^2} & 0 & 0 \\ 0 & 0 & 6 - \frac{4}{N^2} & -\frac{12}{N} \\ 0 & 0 & -\frac{4}{N} & 6 \end{pmatrix} \tag{27}$$

and the exact non-planar corrections to the energy are

$$\begin{aligned}
\delta E_1 &= -\frac{16}{N^2}, & \delta E_2 &= \frac{12}{N^2}, \\
\delta E_{3,4} &= -\frac{2}{N^2} \pm 2\sqrt{\frac{12}{N^2} + \frac{1}{N^4}}.
\end{aligned} \tag{28}$$

Also in this case it turns out that all planar degeneracies are lifted. Obviously, there is another three-excitation sector with one  $W_2$ -excitation and two  $Z_2$ -excitations. The results for that sector can of course easily be read off from those of the present one.

### 6.2.3 Length 8 with 4 excitations

Let us turn to the case of operators of length eight with two excitations of type  $W_2$  and two excitations of type  $Z_2$ . In this sector we find seven operators which descend from the operators treated in section 6.2.1 as well as eight operators which descend from operators

with three excitations. The remaining non-protected operators are

$$\begin{aligned}
\mathcal{O}_1 &= -\text{Tr}(Z_1 W_1 Z_1 W_1 Z_2 W_2 Z_2 W_2) + \text{Tr}(W_1 Z_1 W_1 Z_1 W_2 Z_2 W_2 Z_2) \\
&\quad + \text{Tr}(W_2 Z_1 W_2 Z_1 W_1 Z_2 W_1 Z_2) - \text{Tr}(W_1 Z_2 W_1 Z_1 W_2 Z_1 W_2 Z_2) \\
\mathcal{O}_2 &= \text{Tr}(W_1 Z_2) [\text{Tr}(Z_1 W_2 Z_1 W_1 Z_2 W_2) - \text{Tr}(W_1 Z_1 W_2 Z_1 W_2 Z_2)] \\
&\quad + \text{Tr}(Z_2 W_2) [\text{Tr}(Z_1 W_1 Z_1 W_2 Z_2 W_1) - \text{Tr}(W_2 Z_1 W_1 Z_1 W_1 Z_2)] \\
&\quad + \text{Tr}(Z_1 W_2) [\text{Tr}(Z_1 W_1 Z_2 W_1 Z_2 W_2) - \text{Tr}(W_1 Z_2 W_1 Z_1 W_2 Z_2)] \\
&\quad + \text{Tr}(W_1 Z_1) [\text{Tr}(W_1 Z_1 W_2 Z_2 W_2 Z_2) - \text{Tr}(W_2 Z_1 W_1 Z_2 W_2 Z_2)] \\
\mathcal{O}_3 &= \text{Tr}(W_1 Z_1) \text{Tr}(Z_2 W_2) [\text{Tr}(W_1 Z_1 W_2 Z_2) - \text{Tr}(W_2 Z_1 W_1 Z_2)] \\
&\quad + \text{Tr}(W_1 Z_2) \text{Tr}(Z_1 W_2) [\text{Tr}(Z_1 W_1 Z_2 W_2) - \text{Tr}(W_1 Z_1 W_2 Z_2)] \\
\mathcal{O}_4 &= \text{Tr}(Z_1 W_1 \{Z_1 W_1, Z_2 W_2\} Z_2 W_2) + \text{Tr}(Z_2 W_1 \{Z_2 W_1, Z_1 W_2\} Z_1 W_2) \\
&\quad + \text{Tr}(W_1 Z_1 \{W_1 Z_1, W_2 Z_2\} W_2 Z_2) + \text{Tr}(W_2 Z_1 \{W_2 Z_1, W_1 Z_2\} W_1 Z_2) \\
&\quad - 2\text{Tr}(W_1 Z_1 \{W_1 Z_2, W_2 Z_1\} W_2 Z_2) - 2\text{Tr}(Z_1 W_1 \{Z_1 W_2, Z_2 W_1\} Z_2 W_2) \\
\mathcal{O}_5 &= -\text{Tr}(Z_2 W_2) [\text{Tr}([W_2 Z_1, W_1 Z_1] W_1 Z_2) + \text{Tr}([Z_1 W_1, Z_1 W_2] Z_2 W_1)] \\
&\quad - \text{Tr}(W_1 Z_2) [\text{Tr}([Z_1 W_2, Z_1 W_1] Z_2 W_2) + \text{Tr}([W_1 Z_1, W_2 Z_1] W_2 Z_2)] \\
&\quad - \text{Tr}(Z_1 W_2) [\text{Tr}([Z_1 W_1, Z_2 W_1] Z_2 W_2) + \text{Tr}([W_1 Z_2, W_1 Z_1] W_2 Z_2)] \\
&\quad - \text{Tr}(Z_1 W_1) [\text{Tr}([Z_1 W_2, Z_2 W_2] Z_2 W_1) + \text{Tr}([W_2 Z_1, W_1 Z_2] W_2 Z_2)] \\
\mathcal{O}_6 &= 2\text{Tr}(W_1 Z_1 W_2 Z_2) \text{Tr}(Z_1 W_1 Z_2 W_2) - \text{Tr}(W_2 Z_1 W_1 Z_2) \text{Tr}(Z_1 W_1 Z_2 W_2) \\
&\quad - \text{Tr}(W_1 Z_1 W_2 Z_2) \text{Tr}(W_1 Z_1 W_2 Z_2)
\end{aligned} \tag{29}$$

with planar eigenvalues (in units of  $\lambda^2$ ), trace structure and parity given by

Eigenvector	Eigenvalue	Trace Structure	Parity
$\mathcal{O}_1$	4	(8)	−
$\mathcal{O}_2$	6	(2)(6)	−
$\mathcal{O}_3$	8	(2)(2)(4)	−
$\mathcal{O}_4$	12	(8)	+
$\mathcal{O}_5$	6	(2)(6)	+
$\mathcal{O}_6$	16	(4)(4)	+

We notice one planar parity pair with trace structure (2)(6). In the subspace of negative parity operators the dilatation generator reads

$$\begin{pmatrix} 4 - \frac{12}{N^2} & \frac{12}{N} & \frac{12}{N^2} \\ \frac{12}{N} & 6 & \frac{6}{N} \\ \frac{8}{N^2} & \frac{24}{N} & 8 - \frac{8}{N^2} \end{pmatrix}. \tag{30}$$

The leading corrections to the eigenvalues can be found to be

$$\delta E_1 = -\frac{84}{N^2}, \quad \delta E_2 = -\frac{1728}{N^4}, \quad \delta E_3 = \frac{64}{N^2}. \tag{31}$$

The mixing matrix in the subspace of positive parity eigenvalues looks as follows:

$$\begin{pmatrix} 12 - \frac{12}{N^2} & -\frac{12}{N} & -\frac{8}{N} \\ 0 & 6 & -\frac{8}{N^2} \\ -\frac{72}{N} & 0 & 16 \end{pmatrix}. \tag{32}$$

For these states we find the following leading corrections:

$$\delta E_4 = -\frac{156}{N^2}, \quad \delta E_5 = -\frac{576}{5 N^4}, \quad \delta E_6 = \frac{144}{N^2}. \quad (33)$$

Again we see that all planar degeneracies are lifted.<sup>5</sup>

Summarizing, in all sectors considered we have observed a degeneracy between operators with similar trace structure but opposite parity – a degeneracy which, as explained earlier, could be attributed to the existence of an extra conserved charge and thus to the integrability of the planar dilatation generator. The lift of degeneracies can be taken as an indication (but not a proof) that integrability breaks down beyond the planar level. In any case the concept of integrability when formulated in terms of spin chains and their associated conserved charges has to be reformulated when multi-trace operators are taken into account but it is clear that some symmetries are lost when we go beyond the planar limit.

## 7 BMN operators

In the previous section we analyzed the case of short operators in ABJM theory. Another important class of operators that played a crucial role in the context of the  $AdS_5/CFT_4$  correspondence is that of the so-called BMN operators [26]. It is not difficult to see that BMN operators of ABJM theory can be constructed analogously to BMN operators of  $\mathcal{N} = 4$  SYM [26].

In this section we compute non-planar corrections to the anomalous dimensions of BMN-type operators in the  $SU(2) \times SU(2)$  sector of ABJM theory [15, 16, 17]. We will restrict ourselves to considering BMN operators with two excitations. There are two types of such operators:<sup>6</sup>

$$\mathcal{A}_l^{J_0, J_1, \dots, J_k} = \text{Tr} \left[ Z_2 (W_1 Z_1)^l W_2 (Z_1 W_1)^{J_0-l} \right] \text{Tr} \left[ (Z_1 W_1)^{J_1} \right] \dots \text{Tr} \left[ (Z_1 W_1)^{J_k} \right], \quad (34)$$

$$\mathcal{B}_l^{J_0, J_1, \dots, J_k} = \text{Tr} \left[ (Z_1 W_1)^l Z_1 W_2 (Z_1 W_1)^{J_0-l} Z_1 W_2 \right] \text{Tr} \left[ (Z_1 W_1)^{J_1} \right] \dots \text{Tr} \left[ (Z_1 W_1)^{J_k} \right]. \quad (35)$$

There are in total  $J_0 + 1$  independent operators of type  $\mathcal{A}$  and  $[J_0/2] + 1$  independent operators of type  $\mathcal{B}$ . The associated bare conformal dimensions are

$$\Delta_{\mathcal{A}} = J_0 + \dots + J_k + 1, \quad \Delta_{\mathcal{B}} = J_0 + \dots + J_k + 2. \quad (36)$$

In the spin chain language the  $\mathcal{B}$ -operators have two excitations on the same spin chain whereas the  $\mathcal{A}$ -operators have one excitation on each spin chain. As already mentioned, the  $\mathcal{A}$ -operators do not have an analogue in the scalar sector of  $\mathcal{N} = 4$  SYM<sup>7</sup> where

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<sup>5</sup>However, it is worth noting that the resolution of the degeneracy between  $\mathcal{O}_2$  and  $\mathcal{O}_5$  happens at order  $1/N^4$  and would thus not be visible purely within second order perturbation theory.

<sup>6</sup>As pointed out in [10], these operators resemble scalar operators in the orbifolds of  $\mathcal{N} = 4$  SYM theory in four dimensions. Non-planar corrections for operators in the orbifolded  $\mathcal{N} = 4$  SYM theory have been computed in [37, 38].

<sup>7</sup>This was first pointed out in [19] from the analysis of the dual string theory state.

operators have to organize into representations of  $SO(6)$  (and not into representations of  $SU(2) \times SU(2)$  as here). In  $\mathcal{N} = 4$  SYM two-excitation operators always appear in a symmetrized or anti-symmetrized version.

We wish to study the non-planar corrections to both types of operators. As in  $\mathcal{N} = 4$  SYM we find the set of two-excitation operators above are closed under the action of the dilatation generator, i.e. two-excitation operators with the two excitations in two different traces are never generated when the dilatation generator acts. In the next two sub-sections we consider separately the two sets of operators  $\mathcal{A}_l^{J_0, J_1, \dots, J_k}$  and  $\mathcal{B}_l^{J_0, J_1, \dots, J_k}$ .

Introducing  $J = J_0 + J_1 + \dots + J_k$  we define the BMN limit as the double scaling limit [33, 34]

$$J \rightarrow \infty, \quad N \rightarrow \infty, \quad \lambda' \equiv \frac{\lambda^2}{J^2}, \quad g_2 = \frac{J^2}{N}, \quad \text{fixed.} \quad (37)$$

The BMN limit of the  $\mathcal{N} = 6$  superconformal Chern–Simons–matter theory is expected to correspond to the Penrose limit of the type IIA string theory on  $AdS_4 \times CP^3$ . The string theory states dual to the BMN operators  $\mathcal{A}_l^{J_0, J_1, \dots, J_k}$  and  $\mathcal{B}_l^{J_0, J_1, \dots, J_k}$  have been studied in [17, 19]. Notice, however, that due to different dispersion relations of excitations in the spin chain and string theory language [17] the correct definition of  $\lambda'$  at leading order in a strong coupling expansion is  $\lambda' = \lambda/J^2$  [16, 17].

## 7.1 BMN operators with only one type of excitation

For operators with only one type of excitation the dilatation generator is given by the expression in eqn. (11). Using the notation of eqn. (13) we find

$$D_0 \circ \mathcal{B}_p^{J_0, J_1, \dots, J_k} = -2 \left( \delta_{p \neq J_0} \mathcal{B}_{p+1}^{J_0, J_1, \dots, J_k} + \delta_{p \neq 0} \mathcal{B}_{p-1}^{J_0, J_1, \dots, J_k} - (\delta_{p \neq 0} + \delta_{p \neq J_0}) \mathcal{B}_p^{J_0, J_1, \dots, J_k} \right), \quad (38)$$

$$D_+ \circ \mathcal{B}_p^{J_0, J_1, \dots, J_k} = -4 \left[ \sum_{J_{k+1}=1}^{p-1} \left( \mathcal{B}_{p-J_{k+1}-1}^{J_0-J_{k+1}, J_1, \dots, J_k, J_{k+1}} - \mathcal{B}_{p-J_{k+1}}^{J_0-J_{k+1}, J_1, \dots, J_k, J_{k+1}} \right) \right. \\ \left. - \sum_{J_{k+1}=1}^{J_0-p-1} \left( \mathcal{B}_p^{J_0-J_{k+1}, J_1, \dots, J_k, J_{k+1}} - \mathcal{B}_{p+1}^{J_0-J_{k+1}, J_1, \dots, J_k, J_{k+1}} \right) \right] \quad (39)$$

and

$$D_- \circ \mathcal{B}_p^{J_0, J_1, \dots, J_k} = -4 \left[ \sum_{i=1}^k J_i \left( \mathcal{B}_{J_i+p-1}^{J_0+J_i, J_1, \dots, \mathfrak{J}_i, \dots, J_k} - \mathcal{B}_{J_i+p}^{J_0+J_i, J_1, \dots, \mathfrak{J}_i, \dots, J_k} \right. \right. \\ \left. \left. - \mathcal{B}_p^{J_0+J_i, J_1, \dots, \mathfrak{J}_i, \dots, J_k} + \mathcal{B}_{p+1}^{J_0+J_i, J_1, \dots, \mathfrak{J}_i, \dots, J_k} \right) \right]. \quad (40)$$

The terms resulting from the action of  $D_{++}$ ,  $D_{--}$  and  $D_{00}$  are rather involved and we have deferred them to Appendix B.

We notice that the form of  $D_0$ ,  $D_+$  and  $D_-$  are exactly as for  $\mathcal{N} = 4$  SYM at one loop order, written down in the same notation in [2], except for the fact that  $D_+$  and  $D_-$  in the present case have an additional factor of 2 compared to  $D_0$ . Thus for this type of operators



the analysis up to order  $\frac{1}{N}$  can be directly carried over from [2]. At order  $\frac{1}{N^2}$  one has to take into account the novel terms  $D_{00}$ ,  $D_{++}$  and  $D_{--}$  appearing in Appendix B.1. However, as explained there once one imposes the BMN limit defined in eqn. (37) these terms become sub-dominant. The BMN quantum mechanics is therefore (up to trivial factors of two) identical to that of  $\mathcal{N} = 4$  SYM at one loop level. In particular one encounters the same problem that the huge degeneracies make the perturbative treatment of the non-planar corrections intractable.

## 7.2 BMN operators with two different types of excitations

For operators with two different types of excitations the dilatation generator is given by the expression (11) where we add the similar terms with 1 replaced by 2 and subsequently add the same operator with  $Z$  and  $W$  interchanged. Thus, in this case the dilatation generator consists of 16 terms. Using the notation of eqn. (13) we find

$$D_0 \circ \mathcal{A}_p^{J_0, J_1, \dots, J_k} = -2 \left( \delta_{p \neq J_0} \mathcal{A}_{p+1}^{J_0, J_1, \dots, J_k} + \delta_{p \neq 0} \mathcal{A}_{p-1}^{J_0, J_1, \dots, J_k} - (\delta_{p \neq J_0} + \delta_{p \neq 0}) \mathcal{A}_p^{J_0, J_1, \dots, J_k} \right), \quad (41)$$

$$\begin{aligned} D_+ \circ \mathcal{A}_p^{J_0, J_1, \dots, J_k} = & - \left[ 4 \sum_{J_{k+1}=1}^{p-1} \left( \mathcal{A}_{p-J_{k+1}-1}^{J_0-J_{k+1}, J_1, \dots, J_k, J_{k+1}} - \mathcal{A}_{p-J_{k+1}}^{J_0-J_{k+1}, J_1, \dots, J_k, J_{k+1}} \right) \right. \\ & - 4 \sum_{J_{k+1}=1}^{J_0-p-1} \left( \mathcal{A}_p^{J_0-J_{k+1}, J_1, \dots, J_k, J_{k+1}} - \mathcal{A}_{p+1}^{J_0-J_{k+1}, J_1, \dots, J_k, J_{k+1}} \right) \\ & + 2\delta_{p \neq 0} \left( \mathcal{A}_0^{p, J_1, \dots, J_k, J_0-p} - \mathcal{A}_p^{p, J_1, \dots, J_k, J_0-p} \right) \\ & \left. + 2\delta_{p \neq J_0} \left( \mathcal{A}_{J_0-p}^{J_0-p, J_1, \dots, J_k, p} - \mathcal{A}_0^{J_0-p, J_1, \dots, J_k, p} \right) \right] \end{aligned} \quad (42)$$

and

$$\begin{aligned} D_- \circ \mathcal{A}_p^{J_0, J_1, \dots, J_k} = & -4 \sum_{i=1}^k J_i \left[ \left( \mathcal{A}_{J_i+p-1}^{J_0+J_i, J_1, \dots, J_i, \dots, J_k} - \mathcal{A}_{J_i+p}^{J_0+J_i, J_1, \dots, J_i, \dots, J_k} \right) \right. \\ & \left. - \left( \mathcal{A}_p^{J_0+J_i, J_1, \dots, J_i, \dots, J_k} - \mathcal{A}_{p+1}^{J_0+J_i, J_1, \dots, J_i, \dots, J_k} \right) \right]. \end{aligned} \quad (43)$$

The contributions arising from the action of  $D_{++}$ ,  $D_{--}$  and  $D_{00}$  can be found in Appendix B. Formally  $D_0$ ,  $D_+$  and  $D_-$  are similar to the ones one obtains when applying the one-loop dilatation generator of  $\mathcal{N} = 4$  SYM to an operator containing two different excitations (i.e.  $\Psi$  and  $\Phi$  in a background of  $Z$ 's). The only differences are that the quantities  $D_+$  and  $D_-$  in the present case have an additional factor of 2 compared to  $D_0$  and that there appear two Kronecker  $\delta$ 's in  $D_+$ . However, as already mentioned, in  $\mathcal{N} = 4$  SYM operators with two excitations of different types have to organize into representations of  $SO(6)$  and therefore always come in a symmetrized or anti-symmetrized form. For symmetrized operators, the last line of eqn. (43) vanishes. Taking the BMN limit we observe as before that the terms  $D_{++}$ ,  $D_{--}$  and  $D_{00}$  become sub-dominant, cf. Appendix B.2.

## 8 Conclusion

We have derived and studied the full two-loop dilatation generator in the  $SU(2) \times SU(2)$  sector of  $\mathcal{N} = 6$  superconformal Chern–Simons–matter theory. As opposed to what was the case at leading order in  $\mathcal{N} = 4$  SYM theory, the leading order dilatation generator of ABJM theory implies a mixing not only between  $n$  and  $(n + 1)$  trace states but also between  $n$  and  $(n + 2)$  trace states. The latter mixing becomes sub-dominant when the BMN limit is considered.

By acting with the dilatation generator on short operators we observed at the planar level pairs of degenerate operators belonging to the same representation but having opposite parity. As in planar  $\mathcal{N} = 4$  SYM these degenerate parity pairs could be explained by the existence of an extra conserved charge, the first of the tower of conserved charges of the alternating  $SU(2) \times SU(2)$  spin chain. When non-planar corrections were taken into account these degeneracies disappeared indicating (but not proving) the breakdown of integrability. It would of course be interesting to investigate the mixing problem for higher representations of  $SU(2) \times SU(2)$  than the ones considered here to see if other types of symmetries will reveal themselves. It is clear, however, that once one allows for mixing between operators with different number of traces one needs to re-think the entire concept of integrability. The simple spin chain picture breaks down and the concept of local charges becomes inadequate. In fact, it would be interesting to try to construct a toy example of what one could call an integrable model involving splitting and joining of traces, perhaps along the lines of the simple solvable toy model of reference [29] which describes the splitting and joining of  $\mathcal{N} = 4$  SYM operators dual to the folded Frolov–Tseytlin string [39].

Another interesting and important line of investigation would be to explicitly relate non-planar contributions in the  $\mathcal{N} = 6$  superconformal Chern–Simons–matter theory to observables in the dual type IIA string theory.

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## A Derivation of the non-planar dilatation generator

Here we derive explicitly the full two-loop dilatation generator in the  $SU(2) \times SU(2)$  sector using the method of effective vertices explained in section 3. As already mentioned the scalar D-terms give rise to the following effective vertex

$$\begin{aligned}
(V_D^{bos})^{eff} = \gamma : \text{Tr} & \left[ \left( Z^A Z_A^\dagger + W^{\dagger A} W_A \right) \left( Z^B Z_B^\dagger - W^{\dagger B} W_B \right) \left( Z^C Z_C^\dagger - W^{\dagger C} W_C \right) \right. \\
& + \left( Z_A^\dagger Z^A + W_A W^{\dagger A} \right) \left( Z_B^\dagger Z^B - W_B W^{\dagger B} \right) \left( Z_C^\dagger Z^C - W_C W^{\dagger C} \right) \\
& - 2Z_A^\dagger \left( Z^B Z_B^\dagger - W^{\dagger B} W_B \right) Z^A \left( Z_C^\dagger Z^C - W_C W^{\dagger C} \right) \\
& \left. - 2W^{\dagger A} \left( Z_B^\dagger Z^B - W_B W^{\dagger B} \right) W_A \left( Z^C Z_C^\dagger - W^{\dagger C} W_C \right) \right] :
\end{aligned} \tag{44}$$

where  $: :$  means that self-contractions should be omitted. For the subsequent considerations, it is useful to notice that the following operator gives a vanishing contribution when applied to operators of the type appearing in eqn. (8)

$$\begin{aligned}
V = \gamma \Big\{ \text{Tr} & \left[ \left( Z^A Z_A^\dagger + W^{\dagger A} W_A \right) \left( Z^B Z_B^\dagger - W^{\dagger B} W_B \right) \left( Z^C Z_C^\dagger - W^{\dagger C} W_C \right) \right. \\
& + \left( Z_A^\dagger Z^A + W_A W^{\dagger A} \right) \left( Z_B^\dagger Z^B - W_B W^{\dagger B} \right) \left( Z_C^\dagger Z^C - W_C W^{\dagger C} \right) \\
& - 2Z_A^\dagger \left( Z^B Z_B^\dagger - W^{\dagger B} W_B \right) Z^A \left( Z_C^\dagger Z^C - W_C W^{\dagger C} \right) \\
& - 2W^{\dagger A} \left( Z_B^\dagger Z^B - W_B W^{\dagger B} \right) W_A \left( Z^C Z_C^\dagger - W^{\dagger C} W_C \right) \Big] \\
& - \left[ N \text{Tr} \left( Z_B^\dagger Z^B Z_C^\dagger Z^C \right) - N \text{Tr} \left( Z^B Z_B^\dagger Z^C Z_C^\dagger \right) \right. \\
& + N \text{Tr} \left( W^{\dagger B} W_B W^{\dagger C} W_C \right) - N \text{Tr} \left( W_B W^{\dagger B} W_C W^{\dagger C} \right) \\
& + 2N \text{Tr} \left( Z^B Z_B^\dagger W^{\dagger C} W_C \right) + 2N \text{Tr} \left( W_B W^{\dagger B} Z_C^\dagger Z^C \right) \\
& + 2 \text{Tr} \left( Z^B Z_B^\dagger \right) \text{Tr} \left( Z^C Z_C^\dagger \right) + 2 \text{Tr} \left( W^{\dagger B} W_B \right) \text{Tr} \left( W^{\dagger C} W_C \right) \\
& - 2 \text{Tr} \left( Z^B Z_C^\dagger \right) \text{Tr} \left( Z^C Z_B^\dagger \right) - 2 \text{Tr} \left( W^{\dagger B} W_C \right) \text{Tr} \left( W^{\dagger C} W_B \right) \\
& \left. - 4 \text{Tr} \left( Z^B W_C \right) \text{Tr} \left( Z_B^\dagger W^{\dagger C} \right) \right] \Big\}.
\end{aligned} \tag{45}$$

This can be seen as follows. If we contract  $Z_C^\dagger$  and  $W_C^\dagger$  in the factors  $(Z^C Z_C^\dagger - W^{\dagger C} W_C)$  in the first four lines with  $W$ 's and  $Z$ 's inside the operator  $\mathcal{O}$  we get zero. If we contract the same  $Z_C^\dagger$  and  $W_C^\dagger$  with fields inside the vertex itself we get minus the remaining lines. Notice that there is no normal ordering in the vertex  $V$ .

We can rewrite the above effective vertex (44) in the following way

$$\begin{aligned}
(V_D^{bos})^{eff} = & \gamma \left\{ \text{Tr} \left[ \left( Z^A Z_A^\dagger + W^{\dagger A} W_A \right) \left( Z^B Z_B^\dagger - W^{\dagger B} W_B \right) \left( Z^C Z_C^\dagger - W^{\dagger C} W_C \right) \right. \right. \\
& + \left( Z_A^\dagger Z^A + W_A W^{\dagger A} \right) \left( Z_B^\dagger Z^B - W_B W^{\dagger B} \right) \left( Z_C^\dagger Z^C - W_C W^{\dagger C} \right) \\
& - 2 Z_A^\dagger \left( Z^B Z_B^\dagger - W^{\dagger B} W_B \right) Z^A \left( Z_C^\dagger Z^C - W_C W^{\dagger C} \right) \\
& \left. - 2 W^{\dagger A} \left( Z_B^\dagger Z^B - W_B W^{\dagger B} \right) W_A \left( Z_C^\dagger Z^C - W_C W^{\dagger C} \right) \right] \\
& - : \left[ 3 N \text{Tr} \left( Z^B Z_B^\dagger Z^C Z_C^\dagger \right) + 3 N \text{Tr} \left( Z_B^\dagger Z^B Z_C^\dagger Z^C \right) \right. \\
& + 3 N \text{Tr} \left( W_B W^{\dagger B} W_C W^{\dagger C} \right) + 3 N \text{Tr} \left( W^{\dagger B} W_B W^{\dagger C} W_C \right) \\
& - 2 N \text{Tr} \left( Z^B Z_B^\dagger W^{\dagger C} W_C \right) - 2 N \text{Tr} \left( W_B W^{\dagger B} Z_C^\dagger Z^C \right) \\
& - 2 \text{Tr} \left( Z^B Z_B^\dagger \right) \text{Tr} \left( Z^C Z_C^\dagger \right) - 2 \text{Tr} \left( W^{\dagger B} W_B \right) \text{Tr} \left( W^{\dagger C} W_C \right) \\
& + 12 \text{Tr} \left( Z^B Z_B^\dagger \right) \text{Tr} \left( W^{\dagger C} W_C \right) \\
& - 4 \text{Tr} \left( Z^B Z_C^\dagger \right) \text{Tr} \left( Z^C Z_B^\dagger \right) - 4 \text{Tr} \left( W^{\dagger B} W_C \right) \text{Tr} \left( W^{\dagger C} W_B \right) \\
& \left. - 8 \text{Tr} \left( Z^B W_C \right) \text{Tr} \left( Z_B^\dagger W^{\dagger C} \right) \right] : \\
& - : \left[ 18(N^2 - 1) \text{Tr} \left( Z^C Z_C^\dagger \right) + 18(N^2 - 1) \text{Tr} \left( W^{\dagger C} W_C \right) \right] : \\
& \left. - 24 N^2 (N^2 - 1) \right\}.
\end{aligned} \tag{46}$$

To this effective vertex we must add the effective vertices corresponding to the gluon exchange (Fig. 1b), fermion exchange (Fig. 1c) and scalar self-interactions. What we will get if the “usual” cancellation takes place is the vertex  $V$ . We can rewrite the above vertex without normal ordering as follows

$$\begin{aligned}
(V_D^{bos})^{eff} = & \text{sextic terms} + \text{quartic terms} \\
& + 18(N^2 - 1) \left\{ \text{Tr} \left( Z^C Z_C^\dagger \right) + \text{Tr} \left( W^{\dagger C} W_C \right) \right\} \\
& - 24 N^2 (N^2 - 1).
\end{aligned} \tag{47}$$

Let us continue with the fermion exchange, cf. Fig 1c. It is easy to see that the term  $V_F^{ferm}$  does not contribute to the anomalous dimension of operators of the type (8): A diagram like the one in Fig. 1c requires two fermionic vertices with respectively a daggered and an undaggered scalar field. Such vertices do not appear in  $V_F^{ferm}$ . Furthermore, the first line in  $V_D^{ferm}$  can be shown not to give any contribution. What remains is an effective

vertex which looks like

$$\begin{aligned}
(V^{ferm})^{eff} &= \alpha : \left\{ N\text{Tr} \left( Z^B Z_B^\dagger Z^C Z_C^\dagger \right) + N\text{Tr} \left( Z_B^\dagger Z^B Z_C^\dagger Z^C \right) \right. \\
&\quad + N\text{Tr} \left( W_B W^{\dagger B} W_C W^{\dagger C} \right) + N\text{Tr} \left( W^{\dagger B} W_B W^{\dagger C} W_C \right) \\
&\quad + 4\text{Tr} \left( Z^B Z_B^\dagger \right) \text{Tr} \left( W^{\dagger C} W_C \right) \\
&\quad - 2\text{Tr} \left( Z^B Z_C^\dagger \right) \text{Tr} \left( Z^C Z_B^\dagger \right) - 2\text{Tr} \left( W^{\dagger B} W_C \right) \text{Tr} \left( W^{\dagger C} W_B \right) \\
&\quad \left. - 4\text{Tr} \left( Z^B W_C \right) \text{Tr} \left( Z_B^\dagger W^{\dagger C} \right) \right\} : \\
&= \alpha \{ \text{quartic terms} \\
&\quad - 16(N^2 - 1) \left[ \text{Tr} \left( Z^C Z_C^\dagger \right) + \text{Tr} \left( W^{\dagger C} W_C \right) \right] \\
&\quad + 32N^2(N^2 - 1) \},
\end{aligned} \tag{48}$$

where  $\alpha$  is a coefficient which is to be determined by Feynman diagram computations and where *quartic terms* means the quartic terms from before without normal ordering.

Gluon exchange, cf. Fig 1b gives another contribution to the anomalous dimension of the operators in question. The associated effective vertex reads

$$\begin{aligned}
(V^{gluon})^{eff} &= \beta : \left\{ N\text{Tr} \left( Z^B Z_B^\dagger Z^C Z_C^\dagger \right) + N\text{Tr} \left( Z_B^\dagger Z^B Z_C^\dagger Z^C \right) \right. \\
&\quad + N\text{Tr} \left( W_B W^{\dagger B} W_C W^{\dagger C} \right) + N\text{Tr} \left( W^{\dagger B} W_B W^{\dagger C} W_C \right) \\
&\quad + 2N\text{Tr} \left( Z^B Z_B^\dagger W_C^\dagger W^C \right) + 2N\text{Tr} \left( Z_B^\dagger Z^B W^C W_C^\dagger \right) \\
&\quad + 2\text{Tr} \left( Z^B Z_B^\dagger \right) \text{Tr} \left( Z^C Z_C^\dagger \right) + 2\text{Tr} \left( W^{\dagger B} W_B \right) \text{Tr} \left( W^{\dagger C} W_C \right) \\
&\quad + 4\text{Tr} \left( Z^B Z_B^\dagger \right) \text{Tr} \left( W^{\dagger C} W_C \right) \\
&\quad - 4\text{Tr} \left( Z^B Z_C^\dagger \right) \text{Tr} \left( Z^C Z_B^\dagger \right) - 4\text{Tr} \left( W^{\dagger B} W_C \right) \text{Tr} \left( W^{\dagger C} W_B \right) \\
&\quad \left. - 8\text{Tr} \left( Z^B W_C \right) \text{Tr} \left( Z_B^\dagger W^{\dagger C} \right) \right\} : \\
&= \beta \{ \text{quartic terms} \\
&\quad - 28(N^2 - 1) \left[ \text{Tr} \left( Z^C Z_C^\dagger \right) + \text{Tr} \left( W^{\dagger C} W_C \right) \right] \\
&\quad + 56N^2(N^2 - 1) \},
\end{aligned} \tag{49}$$

where  $\beta$  is a coefficient which likewise is to be determined by Feynman diagram computations.

Noticing that the scalar self-interactions can never give a contribution to the effective vertex which mixes different indices inside the same trace we find that in order that the expected cancellation takes place we need that

$$\alpha = \gamma - 2\beta. \tag{50}$$

Inserting this we find

$$\begin{aligned}
(V_D^{bos})^{eff} + (V^{ferm})^{eff} + (V^{gluon})^{eff} - V = \\
(\beta + 3\gamma)N \left\{ \text{Tr} \left( Z^B Z_B^\dagger \left( W^{\dagger C} W_C - Z^C Z_C^\dagger \right) \right) + \text{Tr} \left( W_B W^{\dagger B} \left( Z_C^\dagger Z^C - W_C W^{\dagger C} \right) \right) \right\} \\
+ (\beta + \gamma)N \left\{ \text{Tr} \left( Z_B^\dagger Z^B \left( W_C W^{\dagger C} - Z_C^\dagger Z^C \right) \right) + \text{Tr} \left( W^{\dagger B} W_B \left( Z^C Z_C^\dagger - W^{\dagger C} W_C \right) \right) \right\} \\
+ (2\beta + 4\gamma) \text{Tr} \left( Z^B Z_B^\dagger - W^{\dagger B} W_B \right) \text{Tr} \left( Z^C Z_C^\dagger - W^{\dagger C} W_C \right) \\
+ (4\beta + 2\gamma)(N^2 - 1) \left\{ \text{Tr} \left( Z^C Z_C^\dagger \right) + \text{Tr} \left( W^{\dagger C} W_C \right) \right\} + (8\gamma - 8\beta)N^2(N^2 - 1). \quad (51)
\end{aligned}$$

As already exploited, terms containing factors of the type  $\left( Z_C^\dagger Z^C - W_C W^{\dagger C} \right)$  only give a non-vanishing contribution when  $Z_C^\dagger$  and  $W^{\dagger C}$  are contracted with fields inside the vertex itself. Therefore, we have

$$\begin{aligned}
(V_D^{bos})^{eff} + (V^{ferm})^{eff} + (V^{gluon})^{eff} - V \\
= (2\beta - 2\gamma)(N^2 - 1) : \left\{ \text{Tr} \left( Z_C^\dagger Z^C \right) + \text{Tr} \left( W_C W^{\dagger C} \right) \right\} : \quad (52)
\end{aligned}$$

This exactly has the form expected for scalar self-interactions. Now we have to determine the coefficients and check that everything fits. From reference [11] we can read off the values of  $\gamma$  and  $\beta$ . They are

$$\gamma = \frac{1}{4} \frac{\lambda^2}{N^2}, \quad \beta = -\frac{1}{8} \frac{\lambda^2}{N^2}. \quad (53)$$

This means that we need that

$$\alpha = \frac{1}{2} \frac{\lambda^2}{N^2}, \quad (54)$$

which can easily be verified using reference [11]. Finally we find for the pre-factor in eqn. (52)

$$(2\beta - 2\gamma)(N^2 - 1) = -\frac{3}{4} \lambda^2 \left( 1 - \frac{1}{N^2} \right). \quad (55)$$

This is exactly equal to minus the pre-factor of the scalar self-energies. The planar part can again be read off directly from [11], while to verify the term subleading in  $N^2$  we performed a closer analysis of the non-planar versions of the self-energy diagrams. Thus, we have shown that the full one-loop dilatation generator in the  $SU(2) \times SU(2)$  sector is indeed given only by the  $F$ -terms in the bosonic potential.

## B Subleading contributions for BMN states

### B.1 Operators with only one type of excitation

Below we present the contributions to  $D\mathcal{B}_p^{J_0, J_1, \dots, J_k}$  which are of order  $\frac{1}{N^2}$ , cf. eqn.(13). As mentioned in the main text none of these terms survive in the BMN limit. As the terms are multiplied by  $\frac{\lambda^2}{N^2}$  they need to be of the order  $J^2$  to contribute in the limit. However,

the maximum order of any term is  $J$ . All terms involve operators in a combination which turns into a first derivative in the BMN limit and which is thus of order  $\frac{1}{J}$ . At the same time any term can at maximum contain two sums (arising via the second and third contraction) each giving a factor of  $J$ .

$$D_{++} \circ \mathcal{B}_p^{J_0, J_1, \dots, J_k} = (-2) \times \quad (56)$$

$$\left[ \sum_{J_{k+2}=1}^{p-J_{k+1}-2} \sum_{J_{k+1}=1}^{p-2} \left( \mathcal{B}_{p-J_{k+1}-J_{k+2}-2}^{J_0-J_{k+1}-J_{k+2}-1, J_1, \dots, J_k, J_{k+1}, J_{k+2}} - \mathcal{B}_{p-J_{k+1}-J_{k+2}-1}^{J_0-J_{k+1}-J_{k+2}-1, J_1, \dots, J_k, J_{k+1}, J_{k+2}} \right) \right. \\ \left. + \sum_{J_{k+2}=1}^{J_0-p-J_{k+1}-2} \sum_{J_{k+1}=1}^{J_0-p-2} \left( \mathcal{B}_p^{J_0-J_{k+1}-J_{k+2}-1, J_1, \dots, J_k, J_{k+1}, J_{k+2}} - \mathcal{B}_{p+1}^{J_0-J_{k+1}-J_{k+2}-1, J_1, \dots, J_k, J_{k+1}, J_{k+2}} \right) \right],$$

$$D_{--} \circ \mathcal{B}_p^{J_0, J_1, \dots, J_k} = \\ -2 \left[ \sum_{i=1}^k J_i \sum_{j \neq i}^k J_j \left( \mathcal{B}_{J_i+J_j+p-1}^{J_0+J_i+J_j, J_1, \dots, \mathfrak{X}_i, \dots, \mathfrak{X}_j, \dots, J_k} - \mathcal{B}_{J_i+J_j+p}^{J_0+J_i+J_j, J_1, \dots, \mathfrak{X}_i, \dots, \mathfrak{X}_j, \dots, J_k} \right. \right. \\ \left. \left. - \mathcal{B}_p^{J_0+J_i+J_j, J_1, \dots, \mathfrak{X}_i, \dots, \mathfrak{X}_j, \dots, J_k} + \mathcal{B}_{p+1}^{J_0+J_i+J_j, J_1, \dots, \mathfrak{X}_i, \dots, \mathfrak{X}_j, \dots, J_k} \right) \right], \quad (57)$$

$$D_{00} \circ \mathcal{B}_p^{J_0, J_1, \dots, J_k} = \quad (58)$$

$$\begin{aligned} & - \left[ 2 \sum_{p=0}^{J_0-1} \left( \mathcal{B}_p^{J_0, J_1, \dots, J_k} - \mathcal{B}_{p+1}^{J_0, J_1, \dots, J_k} \right) + p(p+1) \left( \mathcal{B}_{p-1}^{J_0, J_1, \dots, J_k} - \mathcal{B}_p^{J_0, J_1, \dots, J_k} \right) \right. \\ & + (J_0 - p)(J_0 - p + 1) \left( \mathcal{B}_{p+1}^{J_0, J_1, \dots, J_k} - \mathcal{B}_p^{J_0, J_1, \dots, J_k} \right) \\ & + \sum_{l=0}^{p-1} \left( \mathcal{B}_{p-l-1}^{J_0, J_1, \dots, J_k} - \mathcal{B}_{p-l}^{J_0, J_1, \dots, J_k} \right) + \sum_{l=0}^{J_0-p-1} \left( \mathcal{B}_{p+l+1}^{J_0, J_1, \dots, J_k} - \mathcal{B}_{p+l}^{J_0, J_1, \dots, J_k} \right) \\ & + \sum_{s=0}^{J_0-l-1} \left( \sum_{l=0}^{p-1} + \sum_{l=0}^{J_0-p-1} \right) \left( \mathcal{B}_{l+s+1}^{J_0, J_1, \dots, J_k} - \mathcal{B}_{l+s}^{J_0, J_1, \dots, J_k} \right) \\ & + \sum_{s=0}^{J_0-l-2} \left( \sum_{l=0}^{p-1} + \sum_{l=0}^{J_0-p-1} \right) \left( \mathcal{B}_s^{J_0, J_1, \dots, J_k} - \mathcal{B}_{s+1}^{J_0, J_1, \dots, J_k} \right) \\ & + \sum_{i=1}^k J_i \left( \sum_{l=0}^{J_0-p-1} + \sum_{l=0}^{J_0-p-2} \right) \left( \mathcal{B}_{p+1}^{J_0+J_i-l-1, J_1, \dots, \mathfrak{X}_i, \dots, J_k, l+1} - \mathcal{B}_p^{J_0+J_i-l-1, J_1, \dots, \mathfrak{X}_i, \dots, J_k, l+1} \right) \\ & + \sum_{i=1}^k J_i \left( \mathcal{B}_{p+1}^{p+J_i, J_1, \dots, \mathfrak{X}_i, \dots, J_k, J_0-p} - \mathcal{B}_p^{p+J_i, J_1, \dots, \mathfrak{X}_i, \dots, J_k, J_0-p} \right) \\ & + \sum_{i=1}^k J_i \left( \mathcal{B}_{J_0-p+1}^{J_0-p+J_i, J_1, \dots, \mathfrak{X}_i, \dots, J_k, p} - \mathcal{B}_{J_0-p}^{J_0-p+J_i, J_1, \dots, \mathfrak{X}_i, \dots, J_k, p} \right) \\ & + 2 \sum_{i=1}^k J_i \sum_{l=0}^{J_0+J_i-p-2} \left( \mathcal{B}_{p+1}^{J_0+J_i-l-1, J_1, \dots, \mathfrak{X}_i, \dots, J_k, l+1} - \mathcal{B}_p^{J_0+J_i-l-1, J_1, \dots, \mathfrak{X}_i, \dots, J_k, l+1} \right) \end{aligned}$$

$$\begin{aligned}
& + \sum_{i=1}^k J_i \left( \sum_{l=0}^{p-1} + \sum_{l=0}^{p-2} \right) \left( \mathcal{B}_{p+J_i-l-2}^{J_0+J_i-l-1, J_1, \dots, \mathfrak{X}, \dots, J_k, l+1} - \mathcal{B}_{p+J_i-l-1}^{J_0+J_i-l-1, J_1, \dots, \mathfrak{X}, \dots, J_k, l+1} \right) \\
& + 2 \sum_{i=1}^k J_i \sum_{l=0}^{p+J_i-2} \left( \mathcal{B}_{p+J_i-l-2}^{J_0+J_i-l-1, J_1, \dots, \mathfrak{X}, \dots, J_k, l+1} - \mathcal{B}_{p+J_i-l-1}^{J_0+J_i-l-1, J_1, \dots, \mathfrak{X}, \dots, J_k, l+1} \right) \Big].
\end{aligned}$$

## B.2 Operators with two different types of excitations

Below we present the  $\frac{1}{N^2}$ -contributions to  $D\mathcal{A}_p^{J_0, J_1, \dots, J_k}$ , cf. eqn.(13). As in the case of the  $\mathcal{B}$ -operators and for the same reason none of these terms survive in the BMN limit, cf. Appendix (B.1).

$$\begin{aligned}
D_{++} \circ \mathcal{A}_p^{J_0, J_1, \dots, J_k} = & -2 \left[ \sum_{J_{k+2}=1}^{p-J_{k+1}-2} \sum_{J_{k+1}=1}^{p-2} \left( \mathcal{A}_{p-J_{k+1}-J_{k+2}-2}^{J_0-J_{k+1}-J_{k+2}-1, J_1, \dots, J_k, J_{k+1}, J_{k+2}} - \mathcal{A}_{p-J_{k+1}-J_{k+2}-1}^{J_0-J_{k+1}-J_{k+2}-1, J_1, \dots, J_k, J_{k+1}, J_{k+2}} \right) \right. \\
& - \sum_{J_{k+2}=1}^{J_0-p-J_{k+1}-2} \sum_{J_{k+1}=1}^{J_0-p-2} \left( \mathcal{A}_p^{J_0-J_{k+1}-J_{k+2}-1, J_1, \dots, J_k, J_{k+1}, J_{k+2}} - \mathcal{A}_{p+1}^{J_0-J_{k+1}-J_{k+2}-1, J_1, \dots, J_k, J_{k+1}, J_{k+2}} \right) \\
& + \sum_{J_{k+1}=1}^{p-1} \left( \mathcal{A}_0^{p-J_{k+1}, J_1, \dots, J_k, J_{k+1}, J_0-p} - \mathcal{A}_{p-J_{k+1}}^{p-J_{k+1}, J_1, \dots, J_k, J_{k+1}, J_0-p} \right) \\
& \left. + \sum_{J_{k+1}=1}^{J_0-p-1} \left( \mathcal{A}_{J_0-p-J_{k+1}}^{J_0-p-J_{k+1}, J_1, \dots, J_k, J_{k+1}, p} - \mathcal{A}_0^{J_0-p-J_{k+1}, J_1, \dots, J_k, J_{k+1}, p} \right) \right], \tag{59}
\end{aligned}$$

$$\begin{aligned}
D_{--} \circ \mathcal{A}_p^{J_0, J_1, \dots, J_k} = & -2 \sum_{i=1}^k J_i \sum_{j \neq i}^k J_j \left[ \mathcal{A}_{J_i+J_j+p-1}^{J_0+J_i+J_j, J_1, \dots, \mathfrak{X}, \dots, \mathfrak{Y}, \dots, J_k} - \mathcal{A}_{J_i+J_j+p}^{J_0+J_i+J_j, J_1, \dots, \mathfrak{X}, \dots, \mathfrak{Y}, \dots, J_k} \right. \\
& \left. - \mathcal{A}_p^{J_0+J_i+J_j, J_1, \dots, \mathfrak{X}, \dots, \mathfrak{Y}, \dots, J_k} + \mathcal{A}_{p+1}^{J_0+J_i+J_j, J_1, \dots, \mathfrak{X}, \dots, \mathfrak{Y}, \dots, J_k} \right], \tag{60}
\end{aligned}$$

$$\begin{aligned}
D_{00} \circ \mathcal{A}_p^{J_0, J_1, \dots, J_k} = & - \left[ p(p-1) \left( \mathcal{A}_{p-1}^{J_0, J_1, \dots, J_k} - \mathcal{A}_p^{J_0, J_1, \dots, J_k} \right) \right. \\
& + (J_0-p)(J_0-p-1) \left( \mathcal{A}_{p+1}^{J_0, J_1, \dots, J_k} - \mathcal{A}_p^{J_0, J_1, \dots, J_k} \right) \\
& + 2p \left( \mathcal{A}_{J_0}^{J_0, J_1, \dots, J_k} - \mathcal{A}_{J_0-1}^{J_0, J_1, \dots, J_k} \right) + 2(J_0-p) \left( \mathcal{A}_{J_0-p-1}^{J_0, J_1, \dots, J_k} - \mathcal{A}_{J_0-p}^{J_0, J_1, \dots, J_k} \right) \\
& + 2 \left( \mathcal{A}_{J_0}^{J_0, J_1, \dots, J_k} - \mathcal{A}_0^{J_0, J_1, \dots, J_k} \right) (p \delta_{p \neq 0} + (J_0-p) \delta_{p=J_0}) \\
& + \sum_{l=0}^{p-1} \sum_{s=0}^{J_0-l-2} \left( \mathcal{A}_{J_0-s-1}^{J_0, J_1, \dots, J_k} - \mathcal{A}_{J_0-s-2}^{J_0, J_1, \dots, J_k} \right) + \sum_{l=0}^{p-1} \sum_{s=0}^{J_0-p-l-1} \left( \mathcal{A}_{p-l+s}^{J_0, J_1, \dots, J_k} - \mathcal{A}_{p-l+s-1}^{J_0, J_1, \dots, J_k} \right) \\
& + \sum_{l=0}^{J_0-p-1} \sum_{s=0}^{J_0-l-2} \left( \mathcal{A}_s^{J_0, J_1, \dots, J_k} - \mathcal{A}_{s+1}^{J_0, J_1, \dots, J_k} + \mathcal{A}_{J_0-l-s-2}^{J_0, J_1, \dots, J_k} - \mathcal{A}_{J_0-l-s-1}^{J_0, J_1, \dots, J_k} \right) \\
& \left. + 2 \sum_{i=1}^k J_i \sum_{l=0}^{p-2} \left( \mathcal{A}_{p+J_i-l-2}^{J_0+J_i-l-1, J_1, \dots, \mathfrak{X}, \dots, J_k, l+1} - \mathcal{A}_{p+J_i-l-1}^{J_0+J_i-l-1, J_1, \dots, \mathfrak{X}, \dots, J_k, l+1} \right) \right]
\end{aligned}$$



$$\begin{aligned}
& + 2 \sum_{i=1}^k J_i \sum_{l=0}^{J_0-p-2} \left( \mathcal{A}_{p+1}^{J_0+J_i-l-1, J_1, \dots, \mathfrak{X}_i, \dots, J_k, l+1} - \mathcal{A}_p^{J_0+J_i-l-1, J_1, \dots, \mathfrak{X}_i, \dots, J_k, l+1} \right) \\
& + 2 \sum_{i=1}^k J_i \sum_{l=0}^{J_0-p+J_i-2} \left( \mathcal{A}_{p+1}^{J_0+J_i-l-1, J_1, \dots, \mathfrak{X}_i, \dots, J_k, l+1} - \mathcal{A}_p^{J_0+J_i-l-1, J_1, \dots, \mathfrak{X}_i, \dots, J_k, l+1} \right) \\
& + 2 \sum_{i=1}^k J_i \sum_{l=0}^{p+J_i-2} \left( \mathcal{A}_{p+J_i-l-2}^{J_0+J_i-l-1, J_1, \dots, \mathfrak{X}_i, \dots, J_k, l+1} - \mathcal{A}_{p+J_i-l-1}^{J_0+J_i-l-1, J_1, \dots, \mathfrak{X}_i, \dots, J_k, l+1} \right) \\
& + 2 \sum_{i=1}^k J_i \left( \mathcal{A}_0^{p+J_i, J_1, \dots, \mathfrak{X}_i, \dots, J_k, J_0-p} - \mathcal{A}_{p+J_i}^{p+J_i, J_1, \dots, \mathfrak{X}_i, \dots, J_k, J_0-p} \right. \\
& \left. + \mathcal{A}_{J_0-p+J_i}^{J_0-p+J_i, J_1, \dots, \mathfrak{X}_i, \dots, J_k, p} - \mathcal{A}_0^{J_0-p+J_i, J_1, \dots, \mathfrak{X}_i, \dots, J_k, p} \right) \Big]. \tag{61}
\end{aligned}$$

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